ETERNAL INFLATION

— Alan Guth (MIT)

COSMO-02
Adler Planetarium
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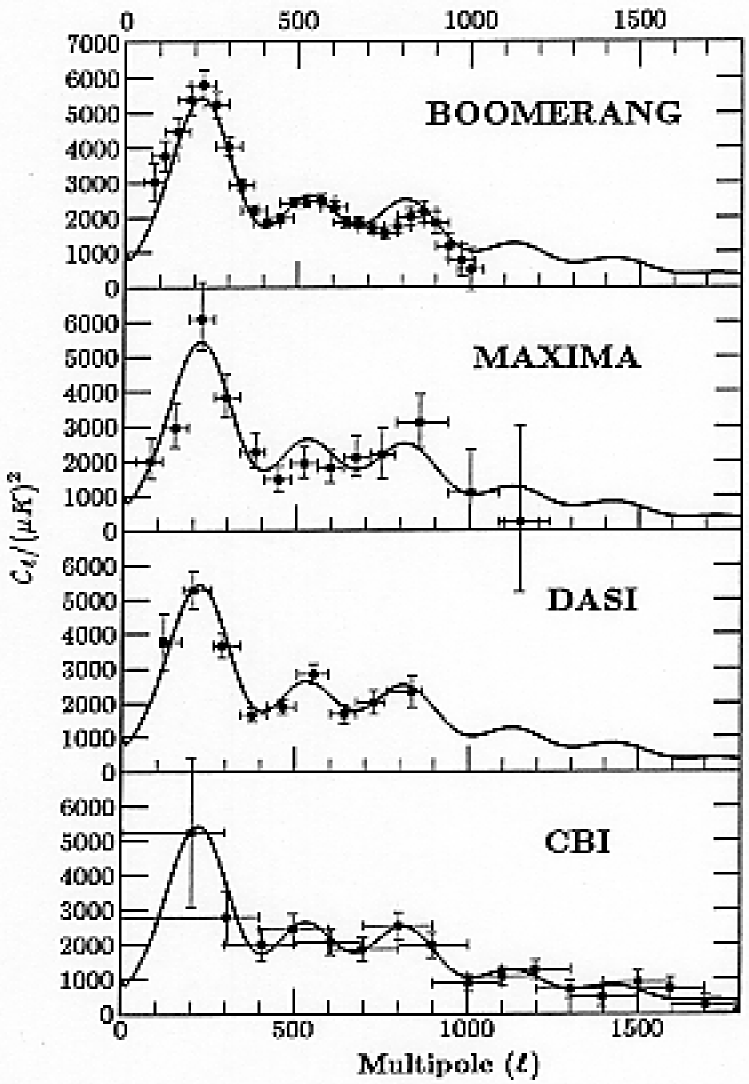
CMB: Cosmic Microwave Background



Parameters of theoretical curve (best fit to all CMB data, CBI group, astro-ph/0205388):

$\Omega_{\rm tot} = 1$ (naturally)

$\Omega_\Lambda = 0.7$ (large dark energy component)

$\Omega_{\rm CDM} = 0.257$ (cold dark matter, consistent with astronomical estimates)

$h = H/(100 \text{ km-sec}^{-1}\text{-Mpc}^{-1}) = 0.68$ (Hubble Key Project: $h = 0.72 \pm 0.08$)

$\Omega_b h^2 = 0.020$ (Big bang nucleosynthesis: $0.020 \pm .001$)

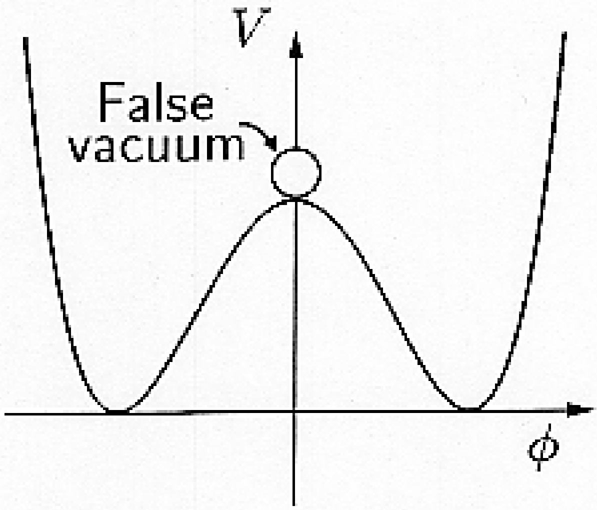
$n_s = 0.95$ (nearly scale-invariant)

$\tau_c = 0$ (no absorption)

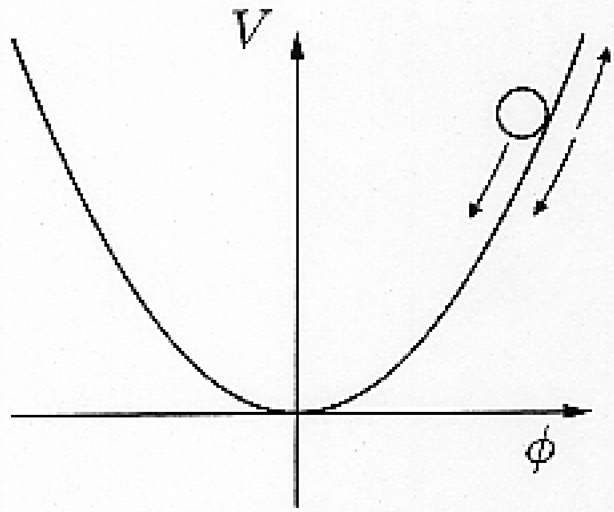
OUTLINE

1. Mechanisms
2. Implications
3. Difficulties
4. Can inflation be past-eternal?

ETERNAL INFLATION



New Inflation



Chaotic Inflation

• New Inflation: False vacuum decays exponentially, but much slower than the exponential expansion. ∴ the volume of false vacuum increases exponentially with time. (Steinhardt, Vilenkin, 1983)

• Chaotic Inflation: Random quantum fluctuations are superimposed on the classical downward motion of the field. In a Hubble time, each Hubble volume expands to $e^3 \approx 20$ Hubble volumes, each of which behaves independently. If $P(\text{upward fluctuation}) > 1/20$, then the volume with $\phi > \phi_{\text{initial}}$ increases with time. (Linde, 1986)

IMPLICATIONS OF ETERNAL INFLATION

1) Hypotheses about initial conditions become divorced from observation. Inflating network presumably approaches steady state.

2) Plausibility of inflation beginning becomes (almost) irrelevant — need only begin once in all eternity. (How could inflation NOT begin!?) In particular, inflation with a scalar field potential of the new inflation type is certainly not dead.

3) Any inflationary scenario that cannot eternally reproduce would seem as implausible as discovering a species of rabbit incapable of reproduction.

4) Inflation could conceivably save the uniqueness of fundamental laws: i.e., String/M-theory may not lead to unique vacuum and unique low-energy physics — but maybe inflation produces overwhelmingly more of one type of vacuum than the others.

DIFFICULTIES WITH ETERNAL INFLATION

Anything that can happen will happen, so a rigorous description of the implications of eternal inflation must be in terms of probabilities. But in the infinite sample volume, the fraction of spacetime with any particular property is infinity/infinity. If one regularizes this quotient, the answer depends on how one chooses the cutoff.

The Youngness Paradox: In an eternally inflating universe, the rate of production of new bubble universes is proportional to the volume of false vacuum. The volume of false vacuum increases exponentially, with a time constant that could be 10^{-37} second. Each second, the production rate of new universes increases by $e^{10^{37}}$. The result is a very youth-dominated "society".

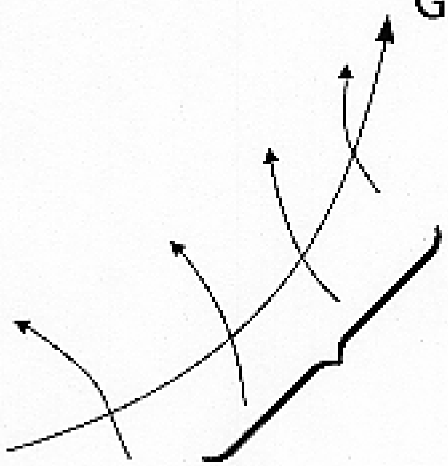
Do we live in the center of the world?, by Linde, Linde, & Mezhlumian, 1994: used an argument, based on "youth-domination," that we most likely live near the center of a spherical hole in the density distribution. Vilenkin and collaborators (Garriga, Tanaka, Vanchurin, Winitzki) have shown that these conclusions can be avoided with an alternative method of calculation, but I believe that it is still unclear how one decides on the right method of calculation.

INITIAL SINGULARITY THEOREM

By Alex Vilenkin, Arvind Borde, and me (2001)

- Previous theorems by Borde & Vilenkin (1994, 1996) depended on the weak energy condition ($n_\mu n_\nu T^{\mu\nu} \geq 0$ for null n^μ, or $\rho + p \geq 0$ for perfect fluids). But the weak energy condition is violated by quantum fluctuations.
- New theorem is purely kinematical. It depends only on the redshifting of velocities in an expanding universe.

Consider a geodesic observer (timelike or null trajectory) moving through an expanding universe:



The observer measures the velocities $u^{\mu}(\tau)$ of the comoving geodesic test particles that she passes, and from their motion she infers a local, unidirectional Hubble parameter

$$H \equiv \frac{\Delta v_{\mathrm{radial}}}{\Delta r} \ .$$

The relative velocity between the test particles and the geodesic observer can be described by

$$\gamma \equiv u_{\mu}\, v^{\mu} = \frac{1}{\sqrt{1 - v_{\mathrm{rel}}^2}} \ ,$$

where the 2nd equality holds only for timelike geodesics.

The redshifting (slowing down) of the relative velocity is directly related to the measurement of H:

$$H = \frac{dF(\gamma)}{d\tau} ,$$

where

$$F(\gamma) = \begin{cases} \gamma^{-1} & \text{for null observers} \\ \frac{1}{2}\ln\left(\frac{\gamma+1}{\gamma-1}\right) & \text{for timelike observers} \end{cases} .$$

$$F(\gamma) = \text{“slowness”} .$$

So, for geodesic observers moving at relative speed $v_{\rm rel}$ at time τ_f,

$$\boxed{\begin{aligned} \int^{\tau_f} H\,d\tau &\le \frac{1}{2}\ln\left(\frac{\gamma+1}{\gamma-1}\right) \\ &= \ln\left(\frac{1}{v_{\rm rel}}\right) + \ln(1+\gamma^{-1}) . \end{aligned}}$$

For null observers, if we normalize the affine parameter τ by $d\tau/dt = 1$ at τ_f, then

$$\boxed{\int^{\tau_f} H\,d\tau \le 1 .}$$

Application to Inflationary Models: In eternally inflating models, the future of any point in the inflating region can be described by a stochastic model for inflaton evolution, valid until the end of inflation. Except for extremely rare large quantum fluctuations, $H \gtrsim \sqrt{(8\pi/3)G\rho_v}$. The past for an arbitrary model is less certain, but we consider eternal models for which the past is like the future. In that case H would be positive almost everywhere in the past inflating region. If, however, $H_{\rm av} > 0$ when averaged over a past-directed geodesic, our theorem implies that the geodesic is incomplete.

The theorem can then be summarized by saying that any spacetime for which $H_{\rm av} > 0$ along any past-directed geodesic cannot be geodesically complete in the past.

Disclaimers: There is of course no conclusion that an eternally inflating model must have a unique beginning, and no conclusion that there is an upper bound on the length of all backwards-going geodesics from a given point. There may be models with regions of contraction embedded within the expanding region that could evade our theorem. Aguirre & Gratton have proposed a model that evades our theorem, in which the arrow of time reverses at the $t = -\infty$ hypersurface, so the universe "expands" in both halves of the full de Sitter space.

Claim: An eternally inflating model of the type usually assumed, which would exhibit global expansion, cannot be complete. Some physics other than inflation would be needed to describe the past boundary of the inflating region. One possibility would be a quantum origin.